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               AN INTERACTION MODEL FOR SUPERSONIC, LAMINAR SEPARATING FLOWS,(U)
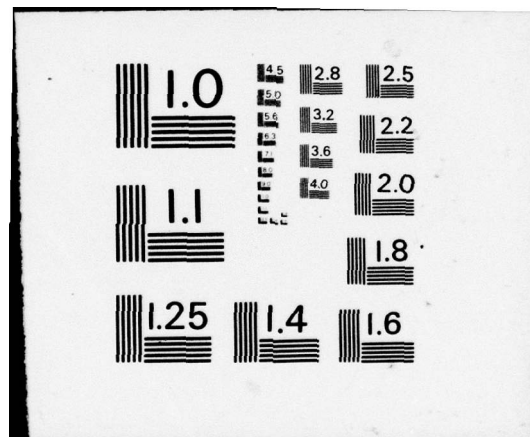   DEC 75   G MILLER   N00014-67-A-0467-0021
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with data are utilized to determine upstream influence properties.   Results
for pressure distribution and heat transfer are presented and compared with
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fields and geometries is assessed.

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AN INTERACTION MODEL FOR SUPERSONIC
LAMINAR SEPARATING FLOWS

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ABSTRACT

An analytical model and computer program has been developed to analyze supersonic, laminar separating flow fields over range. The important features of the model are its ability to include the two family nature of the supersonic flow field, the matching of supersonic and subsonic profiles in the transonic region (which aids in the selection of the solution for a particular problem when two exist, thus eliminating the problem of branching solutions), the inclusion of the normal momentum equation (and thus, normal pressure gradients) throughout the flow field, and the solution of the separation problem by a marching technique. The ellipticity of separated flow fields is accounted for by casting the problem into one of the inverse nature. Correlations with data are utilized to determine upstream influence properties. Results for pressure distribution and heat transffer are presented and compared with experimental data. The applicability/ of the model to more general flow fields and geometries is assessed.

I - INTRODUCTION

The importance of the properties of the boundary layer and its interaction with the external flow field for flows over compression surfaces is well known and the attendant role of these effects on determining aerodynamic characteristics of wing-flap configurations and engine-inlet ramps have been previously assessed. The importance of the viscous-inviscid inter-action is magnified for high Mach number, low Reynolds number flow, and the design of compression surfaces must take into account such inter-action phenomena. Thus, the problem of the interaction of supersonic, laminar flow with a compression surface, and the resulting separated flow field has been the subject of numerous previous experimental (i.e. Ref. 1-5) and analytical investigations.

The flow properties of shock-boundary layer interaction caused either by a deflected flap or by a shock impinging on a boundary layer have been shown to be similar in nature and correlative. The concept of free interaction[6] allows for the employment of empirical relations from a single set of free interaction experimental data to predict the occurence and the extent of a separated zone, as well as the pressure distribution in the region of separation. It has been shown experimentally[7] that in the region of free interaction the heat transfer profiles are independent of the mechanism by which they were induced.

In order to totally quantify the entire flow field, theoretical calculations have had to deal with the basic ellipticity of the problem. The positive pressure disturbance caused by the shock (or by the downstream geo-metry) propagates upstream through the subsonic portion of the boundary

layer causing the boundary layer to separate. Because of the basic elliptic nature, it has been impossible until recently to employ a forward marching scheme without some sort of iteration on the initial conditions.

Many analytical efforts over the past fifteen years have been based on integral techniques [8-11] and have used a moment of conservation equations[12,13] to yield an undetermined parameter which is adjusted to conform with the upstream influence effect. The method of Ref. 13 which uses a one parameter family of profiles, gives good agreement with experimental pressure data for adiabatic and moderately cooled walls, however, for cold walls the method has proved inadequate. Also, the heat transfer results obtained by this method for all wall temperature ratios have shown poor quantitative agreement with experiments. Holden[14] has extended the method of Ref. 13 to allow for separate parameters for the velocity and enthalpy profiles improving agreement with experiment for the highly cooled cases. All of the integral techniques have neglected the possible effects of a normal pressure gradient.

More recent studies[14,15,16] have attempted to use finite difference techniques coupled to an interaction pressure law for the inviscid flow. These solutions have shown a tendency to have an infinite number of branching solutions possible [17,18]. These branched solutions were explained as being caused by various types of downstream geometries causing different rates and magnitues of upstream influence. Dwoyer[16] investigated a variety of branch selections and determined that to control downstream reattachment and relaxation to downstream conditions, it was most convenient to preselect the second derivative of displacement thickness at the initial station.

This parameter could then be adjusted by an iterative procedure until the proper downstream conditions were recovered. All such methods have also neglected the possible effects of normal pressure gradients which may be large in the region of separation and the effects of a two-family supersonic inviscid flow.

A recent investigation[19] has attempted to develop an algorithm which eliminates the necessity of iteration by using the downstream boundary conditions and recasting the interaction equations into a time dependent set. The steady state conditions then corresponds to the physical problem of interest. All previous studies, whether utilizing finite difference or integral techniques have had to assume a pressure-streamline deflection relationship at the boundary layer edge (either a linear dependence or a tangent wedge approximation), to determine interaction effects and thus have of necessity, neglected the effects of embedded shocks, and the attendant effects of downrunning waves which can alter boundary layer properties. In addition, whereas a linear pressure-deflection relationship applies for weak interaction and the tangent wedge relationship approximates strong interaction, it is always a problem to decide for a particular problem which relationship to use, especially in the overlap domain (i.e. $X \sim O(1)$).

Previous work at New York University[20,21,22] has attempted to establish a unified approach for the solution of expanding and compressing flows where the effects of interaction are important. The investigations have sought to demonstrate the validity of the model developed (which will be

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described in the next section) and the general applicability of the numer-
ical analysis which has been produced. The important features of the
model are: its ability to include the two family feature of the super-
sonic field (which all other previous analyses have neglected), the match-
ing of supersonic and subsonic profiles in the transonic region (which aids
in the selection of the solution for a particular problem when two exist,
thus eliminating the problem of branching solutions) and the inclusion of
the normal momentum equation (and thus normal pressure gradients) in the super-
sonic field (by the use of the viscous characteristic equations) and in the
highly viscous subsonic zone (solved by an implicit finite difference scheme).
It has thus been asserted[21] that interaction problems can be well posed as
initial value problems within the context of this model.

The ellipticity of separating and reattaching flows over ramps adds on addi-
tional parameter to the general description of the model, which is based
on a marching scheme. The effects of strong upstream influence must be
accounted for and the following sections delineate how this is handled.
Following a section describing the model and its special details necessary
for separated flow fields, a section on the equations of motion in each of
the regions of the problem is presented. Then, results of the analysis for
two separated ramp flow fields are described. The results are compared with
experiments, and conclusions of the model's applicability to separated
flow fields is presented.

II. DESCRIPTION OF THE PROBLEM AND THE ANALYTICAL MODEL

The basic problem which is studied is a supersonic or hypersonic flow over a wing-flap configuration. As an example of this phenomenon consider a flow over a flat plate-wedge configuration shown schematically in Fig. 1. The start of the free interaction region occurs at the station (X_O) where the wall properties begin to depart from flat plate values. In this region, an adverse pressure gradient is communicated through the subsonic portion of the boundary layer, causing the wall pressure coefficient to rise while simultaneously, the skin friction and heat transfer decrease. Outgoing compression waves begin to feed upward into the freestream. For supersonic Mach numbers, this compression fan coalesces into a shock some distance away from the wall. In the hypersonic case, the shock forms almost at the boundary layer edge (or near the subsonic-supersonic viscous flow interface), its slope approximating the flow deflection of the boundary layer edge itself. In both cases the flow downstream of the coalescence may be dominated by a two family characteristic flow field as downrunning waves from the shock may significantly alter the pressure distribution given by a Prandtl-Meyer relation. The separation point (X_S) is defined by the point of vanishing skin friction. A dividing stream-line can be identified which forms the boundary between positive and negative values of the stream function. A second compression fan and shock are found due to the reattachment compression.

Figure 2 depicts schematically the wall pressure, heat transfer and skin friction distributions. In the region of free interaction the pressure gradient is approximately constant and then decreases gradually until a pressure plateau is reached. In the region of the flap the pressure again

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begins to rise, reaching a maximum and then decreasing asymptotically to the two-dimensional wedge value. The skin friction after the hinge increases from a negative value, passes through zero at reattachment and increases again to the flat plate value. The heat transfer distribution reaches its minimum in the region of the hinge and then increases as shown in the figure.

The analytical model and method of analysis developed in[20,21,22] for expanding flow fields has been modified to allow for the analysis of the above problem. The method of analyses is as follows: In the region prior to the effect of upstream influence the method described in Ref. 21 is used unaltered. The supersonic portion of the flow field is solved by characteristics which have been modified to include viscous transports normal to the streamlines. The inclusion of viscosity in the characteristic relations allows for an inner-outer matching to occur in the transonic region. The equations for the inner region (the highly viscous subsonic flow near the wall) are represented by the boundary layer equations in coordinates measured along and normal to the surface. The dictate from the outer flow field that normal pressure gradients may occur, coupled with the effect of the thickening boundary layer (enhanced for moderate Reynolds numbers and/or the onset of separation) leads to the inclusion of normal pressure gradient effects in the inner flow field. Thus, the second momentum equation is retained in the inner region. The inner equations are solved by an implicit finite difference Crank-Nicholson scheme. The matching between inner and outer solutions is accomplished by guessing a streamline deflection at the matching point of the next step (whose stepsize is dictated by the characteristic intersection distance) and determining the pressure at the transonic intersection from the downrunning characteristic relation. The inner

region is then computed and the condition that at the wall the stream function is zero is tested. An iteration procedure based on changing the streamline deflection is continued until the surface condition ($\Psi_{wall} = 0$) is met.

The interaction program is then applied to the separation problem in an inverse manner. At a selected station along the flat plate, a selected adverse pressure gradient is applied at the wall (The method for the selection of Xo and $\frac{dp}{dx}$ is explained in Section IV.) Consistent with the thus known wall pressure, the iteration procedure is dropped. This is equivalent to allowing for mass injection at the surface (since $\Psi_{wall} \neq 0$). It has previously been shown[23] that there is a close correspondence between the upstream influence of a downstream flap and a boundary layer with injection. This procedure is continued until flow separation. At the separation point, the complete iteration procedure is again reverted to (it is no longer necessary to specify the adverse pressure gradient) and the system is solved consistent with $\Psi_{wall} = 0$. An important result is that the pressure gradient which is determined in the course of the marching is entirely consistent with the imposed adverse pressure gradient prior to separation. It is the assertion here that the pressure gradient is continuous after separation due to the fact that the supersonic characteristic field in the vicinity of the matching has accommodated to the effect of streamline curvature and thus the downrunning characteristic (which determines the pressure) is consistent with the previous pressure distribution. An embedded shock routine has been added to the main program to allow for characteristic coalescence when it occurs (See Fig. 1).

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It has been shown [14,16] that instabilities can arise in the numerical
solution of the boundary layer equations by a marching technique in
regions of reverse flow. This arises from the fact that the convective
terms $\rho u u_x$ and $\rho u T_x$ in the momentum and energy equations become nega-
tive. These instabilities also arose in the present analysis in the
region of separation. To overcome this effect, the same approach as
has been used in previous investigations was employed, i.e, the utili-
zation of the absolute value of u in the convective terms
$(\rho|u|u_x$ and $\rho|u|T_x)$.

At a certain previously unknown station, the program is no longer able
to converge on a solution (i.e., pass the mass flow). At this station
the wall is turned. The position of and the angle of the turn which
were previously unknown are the two parameters which are now determined,
thus rendering the program one of an inverse nature. Thus, the ellipticity
of the flow field is overcome by determining the surface geometry which
caused the particular upstream separating pressure gradient (and its posi-
tion) as a consequence of the solution. The wall turning is accomplished
within the program by turning the wall through a small $\Delta\theta$ in each step Δx
until it is no longer necessary to change θ_w to pass the mass flow. Thus,
the angle of wall turning is determined and the program is continued
through reattachment.

Thus, the flow field has been separated into six distinct regions of compu-
tation (Fig. 3) .

    I. Region prior to upstream influence

   II. Region of prescribed adverse pressure gradient

III. Region of separated flow where convective terms are utilized with absolute value of u in the negative region

IV. Region of wall turning (dictated by the inability to have a converged solution)

V. Region approaching reattachment along wedge (solved in same way as region III)

VI. Region of attached flow relaxing to Blasius solution

III. EQUATIONS OF MOTION

The equations of motion used in the distinct regions of the flow field have been presented previously [20-22]. In the supersonic portion of the flow field viscous transports normal to the streamlines are included in the characteristic yielding the following two-dimensional equations for the determination of the properties:

$$\frac{dp}{\gamma p} \pm \frac{d\,\theta}{\sin\mu\cos\mu} + \left[\frac{(S_1)}{Pq^2} - \frac{\gamma-1}{\gamma}\frac{S_2}{pq}\right]\frac{dx}{\cos\mu\cos(\theta+\mu)} = 0$$

where

$$S_1 = \frac{1}{Re_\infty}\left(\frac{\partial}{\partial n}\left(\bar{\mu}\,\frac{\partial q}{\partial n}\right)\right)$$

$$S_2 = \frac{1}{Re_\infty}\left[\bar{\mu}\left(\frac{\partial q}{\partial n}\right)^2 + \frac{1}{PrE_\infty}\frac{\partial}{\partial n}\left(K\,\frac{\partial T}{\partial n}\right)\right]$$

(1)

The nomenclature is consistent with the previous references. The plus sign represents the uprunning characteristic, the minus sign the downrunning one along which:

$$\frac{dy}{dx} = \tan(\theta\pm\mu)$$

(2)

The inclusion of viscosity in the net allows for the matching between inner and outer solutions to occur in the transonic region. In the inner region, the standard boundary layer equations in coordinates measured along and normal to the surface are utilized. Terms of the order of $1/\sqrt{Re_\infty}$ are retained allowing for moderate Reynolds number effects. The normal momentum equation is retained. The equations are thus (for two-dimensional flows with

curvature terms neglected)

$$(\rho u)_x + (\rho \bar{v})_{\bar{y}} = 0 \tag{3}$$

$$\rho(u u_x + \bar{v} u_{\bar{y}}) + P_x = S_1' \tag{4}$$

$$\rho\left(\frac{u\bar{v}_x}{\sqrt{Re_\infty}} + \frac{\bar{v}\bar{v}_{\bar{y}}}{\sqrt{Re_\infty}}\right) + P_{\bar{y}}\sqrt{Re_\infty} = 0 \tag{5}$$

$$\rho(u T_x + \bar{v}\, T_{\bar{y}}) - E_\infty(u P_x + \bar{v}P_{\bar{y}}) = S_2' \tag{6}$$

where $S_1' = (\bar{\mu}\, u_{\bar{y}})_{\bar{y}}$

$$S_2' = \frac{1}{Pr}\,(\bar{\mu}T_{\bar{y}})_{\bar{y}} + E_\infty\bar{\mu}\,(u_{\bar{y}})^2$$

The solution of the system of equation and method of iteration valid in each region of the flow field has been described in the previous section. One additional statement must be made with respect to the solution of the continuity equation for v in the subsonic zone. The matching of solutions in the transonic zone causes one to be compelled to use a small stepsize (since in the transonic zone characteristics are close to vertical). This small stepsize has caused an instability in the determination of v from continuity when it is computed from the equation $\rho v = -\frac{\partial\psi}{\partial x}$. This has been overcome consistantly for both expanding and compressing flow fields by utilizing the modified equation $\rho v = -\frac{\partial\psi}{\partial \overline{X}}$, where $\overline{\Delta X}$ is taken to be on the order of ten ΔX and then decreased during the course of the marching. In the following section results of the computer program are presented and comparisons with experiments are shown to demonstrate the validity of the model.

IV. RESULTS AND CONCLUSIONS

The basic ellipticity of the problem necessitates a determination of two parameters, the point of demarkation from the flat plate results (Xo) and the pressure gradient from there to the separation point $\left(\frac{dp}{dx}\right)_{Xo}$, in order to proceed with the calculation. The problem is thus cast as one of an inverse nature, i.e., select and Xo and $\frac{dp}{dx}$ and find the plate length L and wall turn θ_w which brought about the upstream effect. In order to calculate the flow for a particular problem it is thus necessary to iterate on the initial conditions until the two parameters are consistent with the flow of interest.

In order to somewhat overcome this complexity, and large increase in computer time per case, an analysis of available data was undertaken to correlate results for these parameters over a wide range of the interaction parameter χ.

Based on the suggestions of[24,25], Cheng[26] found that the important parameter governing the similitude of the flow at separation is $(M_\infty \theta \text{ incip})^2/\chi_\epsilon$ where θincip is the ramp angle at incipient separation and $\chi_\epsilon = \frac{\gamma-1}{\gamma+1}$ $(.664 + 1.73\,Tw/T_0)\chi$. This effect was correlated over a wide range from weak to strong interaction. It was also shown that the upstream influence in an attached flow was inversely proportional to $M_\infty/\sqrt{Re_L}$.

Based on these conclusions, the data of separated flow fields was investigated to determine the correlativity of $\frac{Xo}{L}$ and $\left(\frac{dp/p_\infty}{dx/L}\right)_{Xo}$ as a function of χ_ϵ. It was found that for $\theta_w > \theta$incip (separated flows), the parameter $\frac{M_\infty \theta +,\, Xo/L}{\chi_\epsilon^{1/2}}$ governs the similitude of the flow and that $\frac{dp/p_\infty}{d(x/L)}$ in the region $Xo < X < X_s$ also varies as $\chi_\epsilon^{1/2}$. These results are presented in Figures

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4 and 5, which present data over a wide range of the interaction param-
eter.

These correlations thus allow for the selection of the two parameters Xo and
$\frac{dp}{dx}$ for any flow condition and ramp angle with reasonable accuracy. The iter-
ation procedure to determine the exact values for a particular geometry is
thus reduced considerably and the inverse characteristic of the model no
longer produces a major drawback.

The results of two calculations corresponding to the data of Ref. 2 are pre-
sented here. For these cases, the correlations utilized allowed that no
iteration on initial conditions were needed to find the flow fields consis-
tent with the cases to be reproduced. The results for wall pressure and
heat transfer, presented in Figures 6-8, indicate that the model reproduces
the flow phenomena with good accuracy.

It is thus the contention that separated flow fields in the supersonic and
hypersonic regimes can be computed utilizing the interaction model presented
herein, which computes the entire flow at each step of a marching technique
without the necessity of specifying a pressure-deflection relationship and
allowing for embedded shocks and the two family nature of the flow field.
The inverse nature of the analysis is somewhat overcome for ramps by utilizing
the correlations developed. This, of course, would not be the case for more
complex geometries. The major drawback of this model is the necessity for
a given flow condition, to have an available correlation for the determination
of the upstream influence effect. Therefore, while in principle the model
has general applicability for the solution of laminar separated flowfields
in the supersonic and hypersonic regime, in practice, complex geometric con-
figurations (including those which yield three-dimensional flowfields) may be

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more easily accommodated within the scope of fully elliptic programs.

With respect to turbulent flows, one could foresee a similar situation. Ample experimental data is available for turbulent flows over ramps to develop a correlation similar to the ones presented in Figures 4 and 5. One would expect that the correlations would display a different power law than in the laminar case, perhaps yielding solutions which vary as x^{-8}.
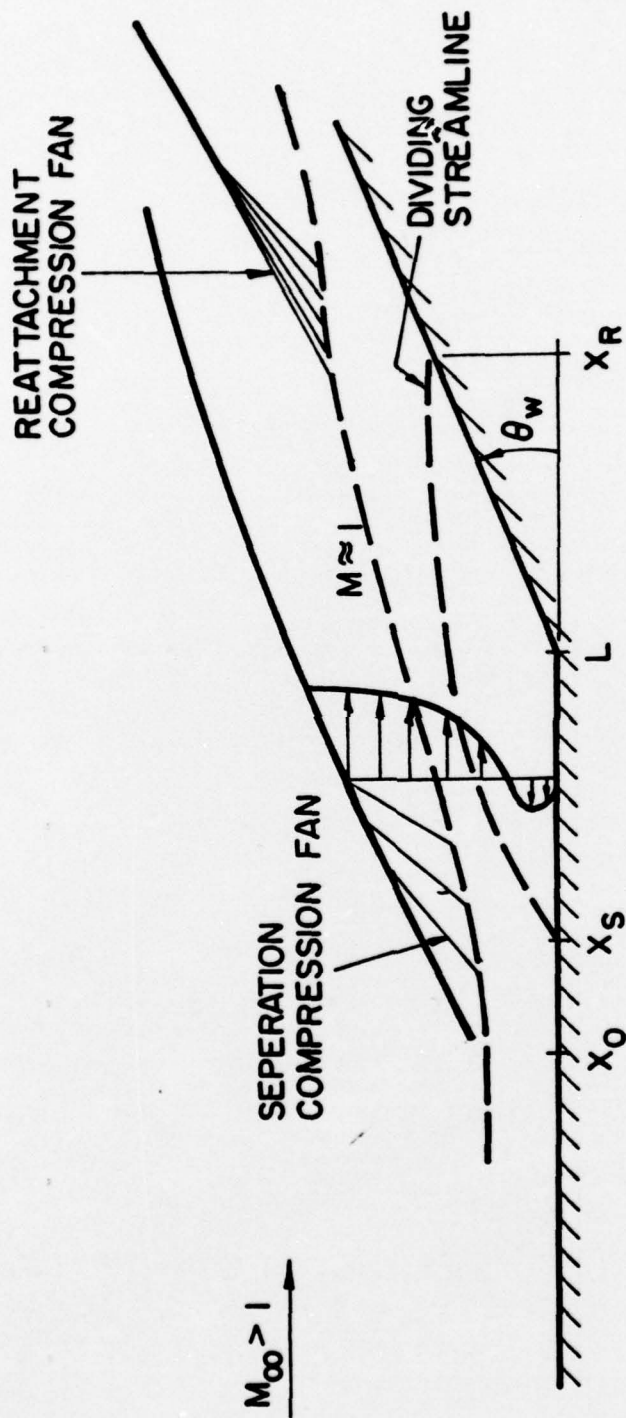
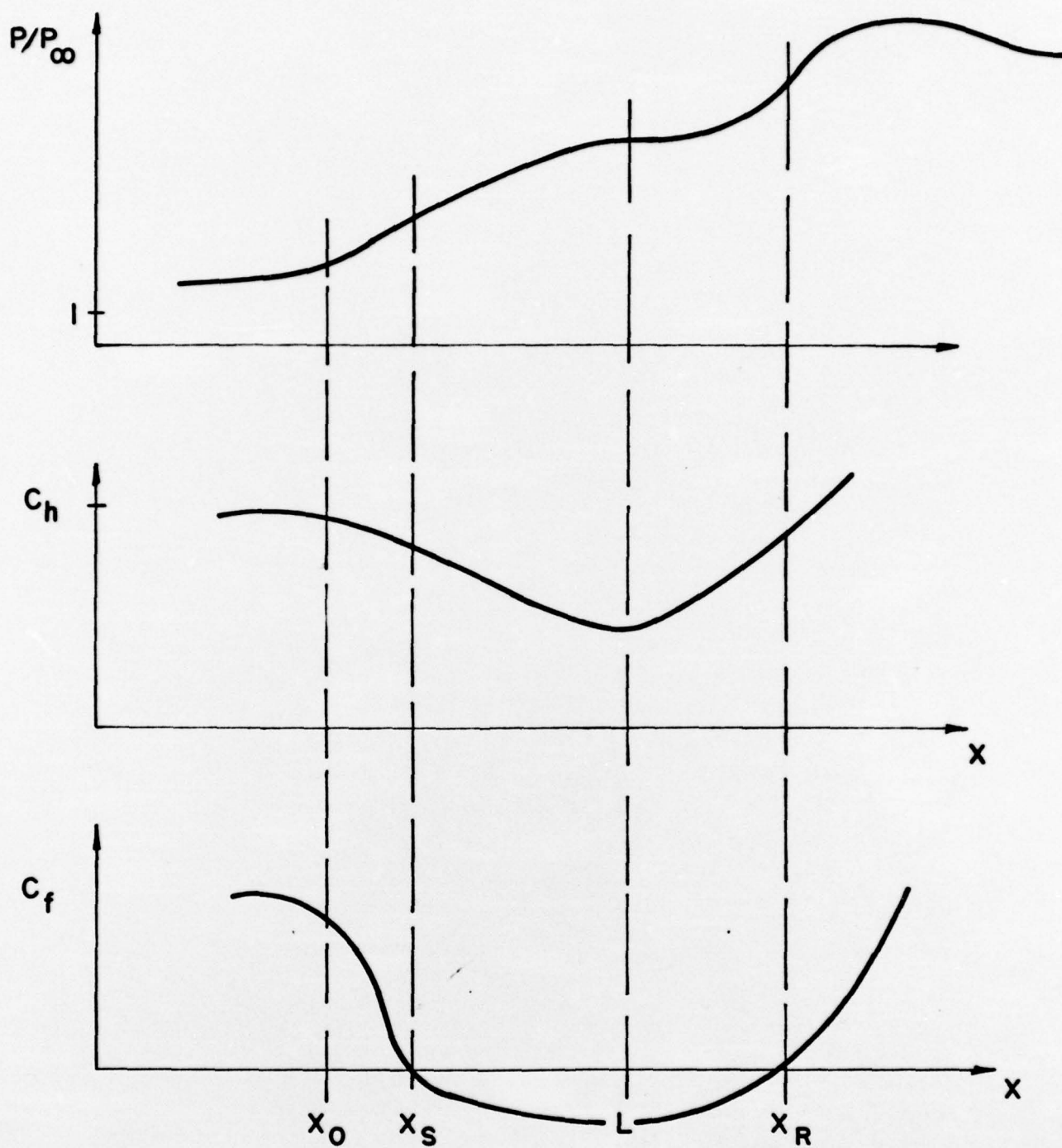FIGURE 1. SCHEMATIC REPRESENTATION OF SUPERSONIC FLOW OVER A RAMP

FIGURE 2. SCHEMATIC REPRESENTATION OF PRESSURE, HEAT TRANSFER AND SKIN FRICTION DISTRIBUTIONS IN SEPARATED FLOW OVER RAMP
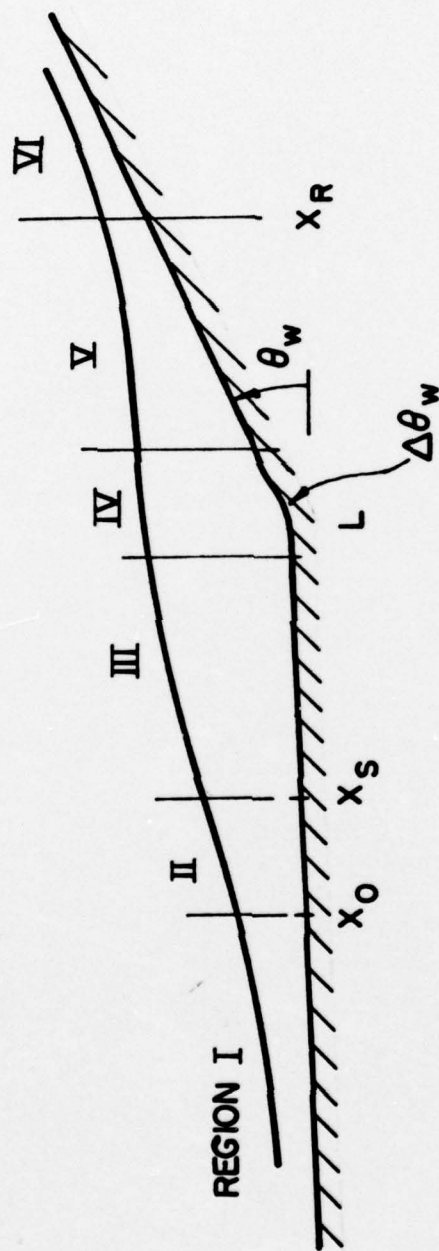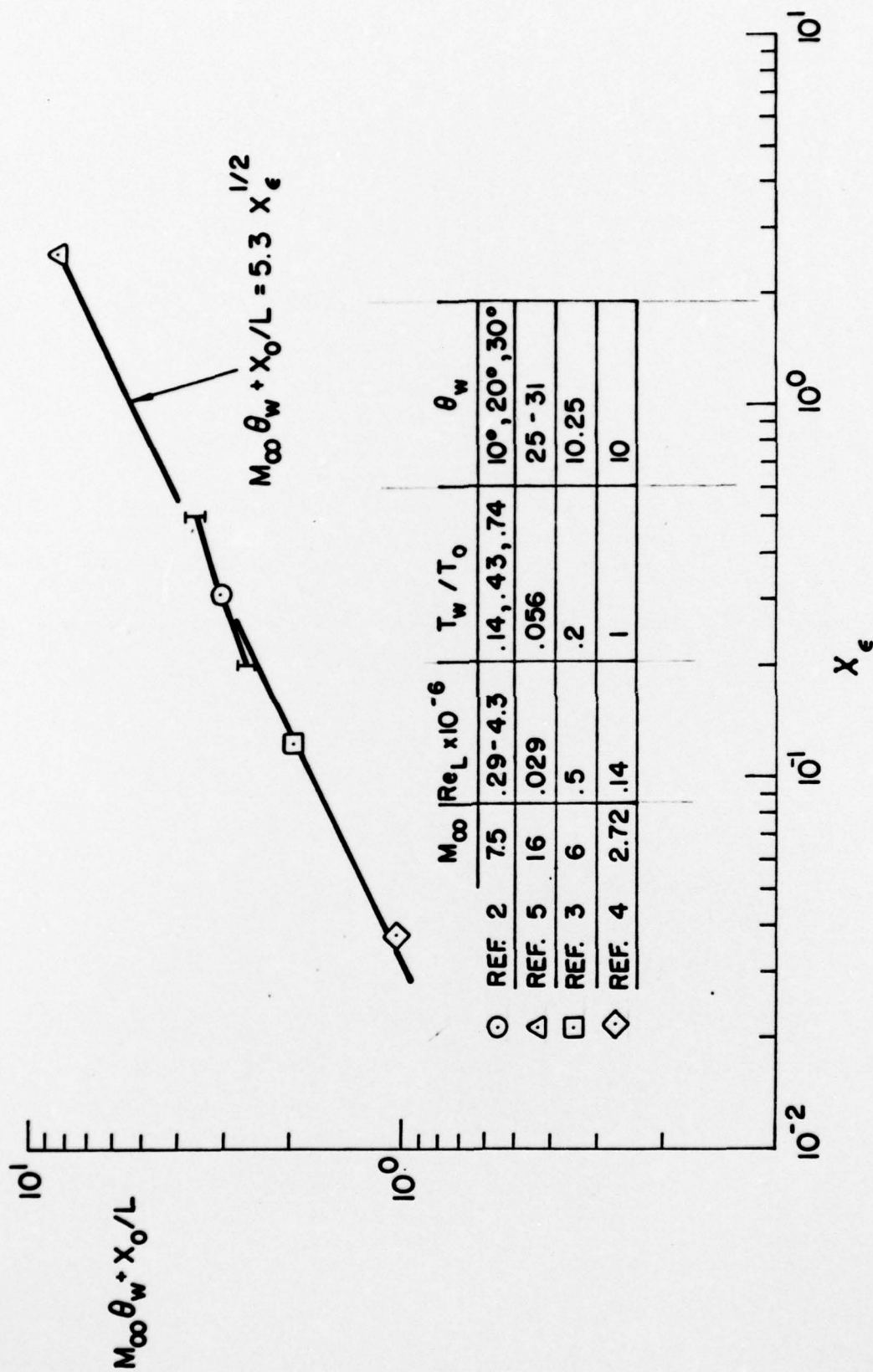
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FIGURE 3. DISTINCT REGIONS OF COMPUTATION

M_∞	$Re_L \times 10^{-6}$	T_w/T_o	θ_w
⊙ REF. 2	7.5	.29 - 4.3	.14, .43, .74	10°, 20°, 30°
△ REF. 5	16	.029	.056	25 - 31
▫ REF. 3	6	.5	.2	10.25
◇ REF. 4	2.72	.14	1	10

$$M_\infty \theta_w + x_o/L = 5.3\ \bar{x}_\epsilon^{1/2}$$

FIGURE 4. CORRELATION OF DATA IN TERMS OF INTERACTION PARAMETER TO DETERMINE SEPARATION x_o/L
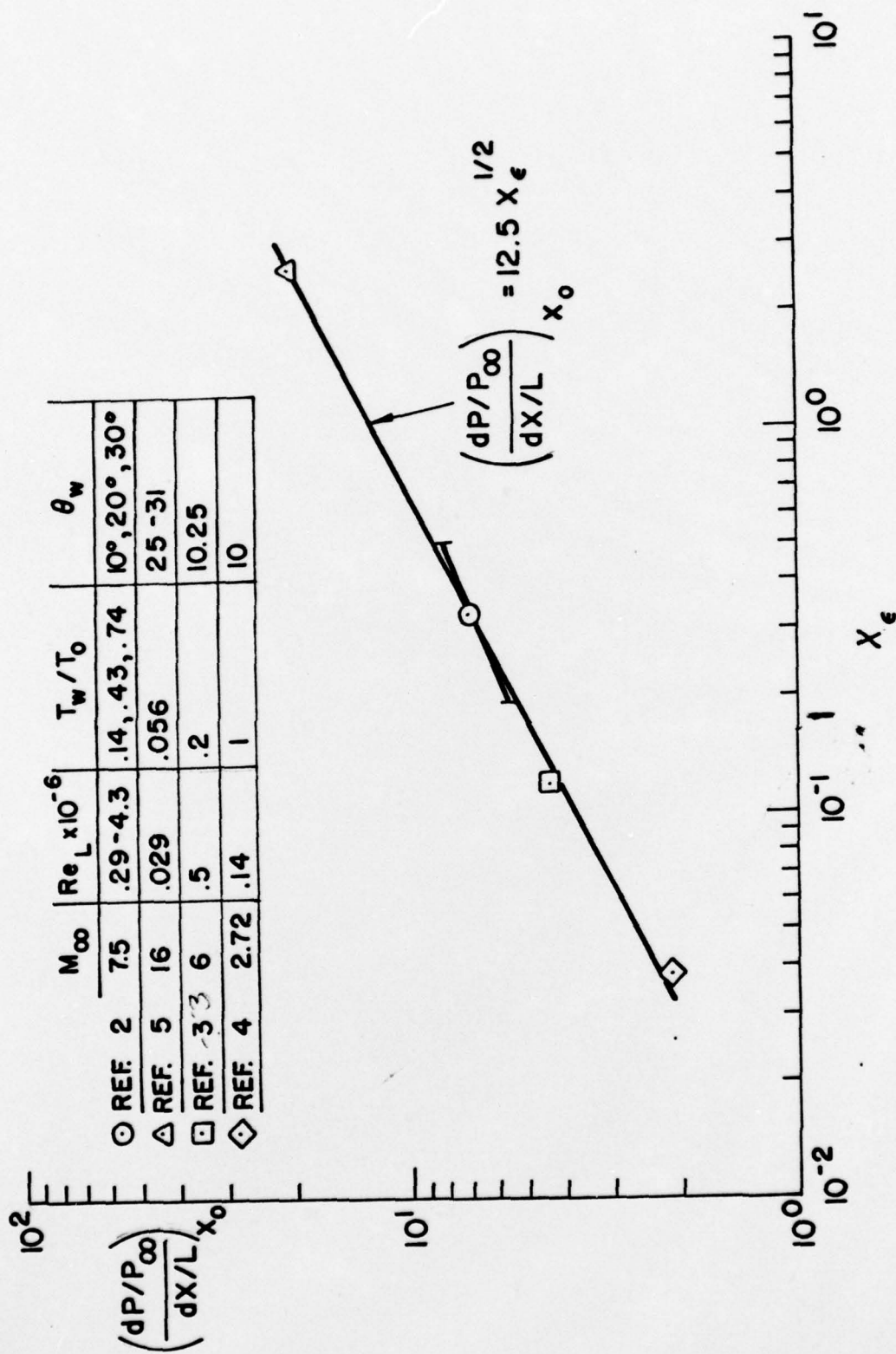
FIGURE 5. CORRELATION OF DATA IN TERMS OF INTERACTION PARAMETER TO DETERMINE SEPARATION PRESSURE GRADIENT
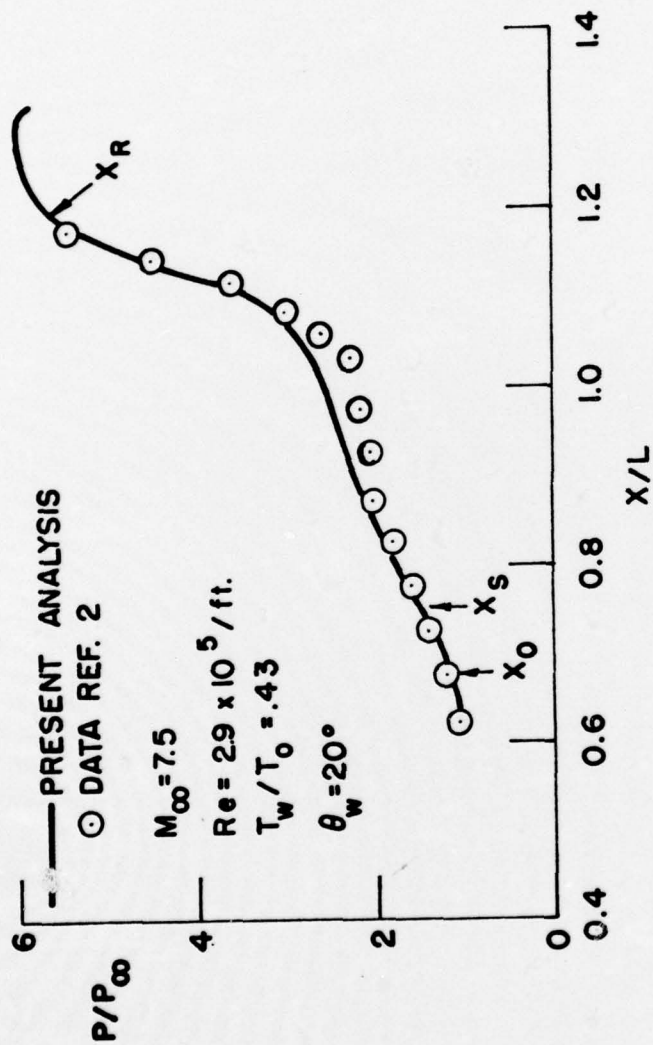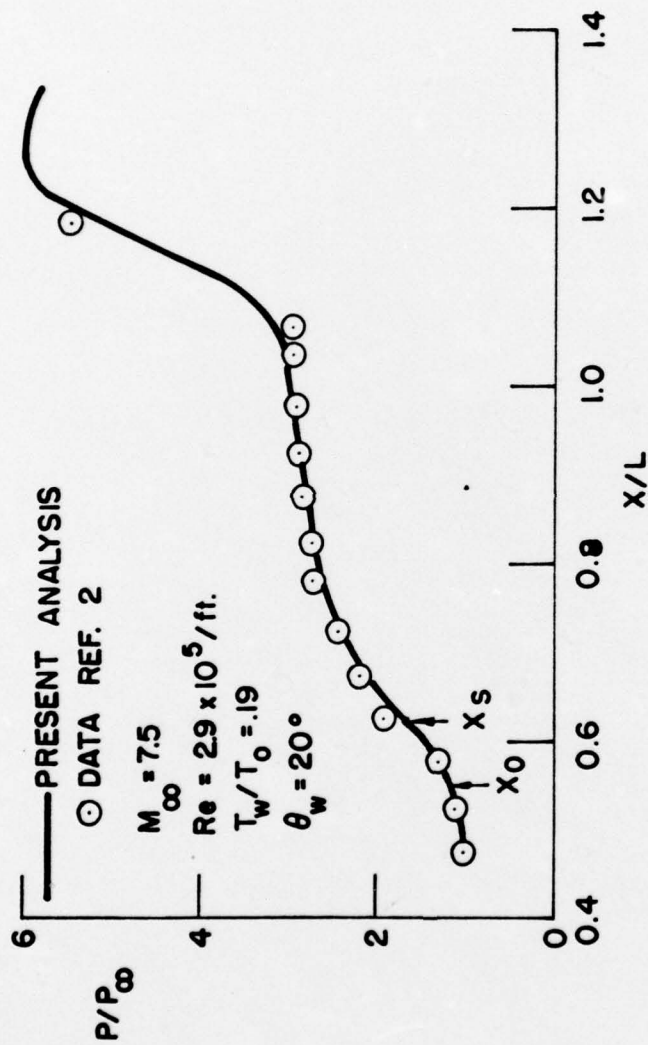
FIGURE 6. PRESSURE DISTRIBUTION IN A SEPARATED FLOW FIELD AND COMPARISON WITH DATA OF REF. 2.

($M_\infty - 7.5$, $Re = 2.9 \times 10^5$/ft., $T_W/T_0 = .43$, $\theta_W = 20°$)

FIGURE 7. PRESSURE DISTRIBUTION IN A SEPARATED FLOW FIELD AND COMPARISON WITH DATA OF REF. 2.
(M_∞ = 7.5, Re = 2.9 x 10^5/ft., Tw/To = .19, θ_w = 20°)

PRESENT ANALYSIS
⊙ DATA REF. 2
$M_\infty = 7.5$
$Re = 2.9 \times 10^5 /ft.$
$T_w/T_0 = .43$
$\theta_w = 20°$

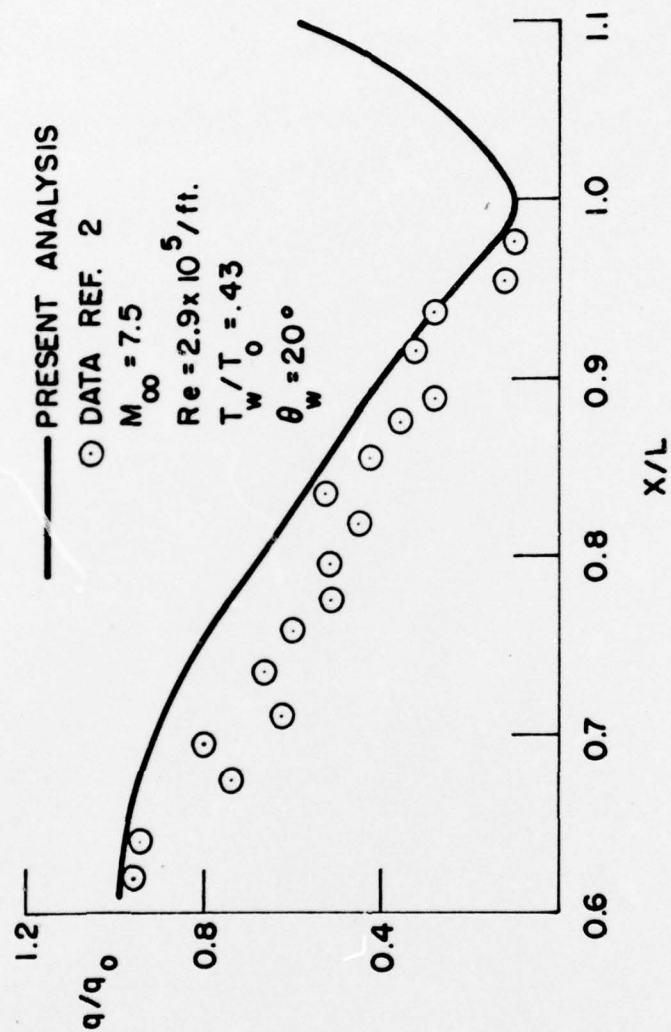q/q_0

1.2

0.8

0.4

X/L

0.6　0.7　0.8　0.9　1.0　1.1

FIGURE 8.　HEAT TRANSFER DISTRIBUTION IN A SEPARATED FLOW FIELD AND COMPARISON WITH DATA OF REF. 2.

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